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Exact results for non-holomorphic masses in softly broken supersymmetric gauge theories

Nima Arkani-Hamed^a and Riccardo Rattazzi^b

^a *Stanford Linear Accelerator Center*

Stanford University

Stanford, California 94309, USA

^b *Theory Division, CERN*

CH-1211, Genève 23, Switzerland

Abstract

We consider strongly coupled supersymmetric gauge theories softly broken by the addition of gaugino masses m_λ and (non-holomorphic) scalar masses m^2 , taken to be small relative to the dynamical scale Λ . For theories with a weakly coupled dual description in the infrared, we compute exactly the leading soft masses for the “magnetic” degrees of freedom, with uncalculable corrections suppressed by powers of (m_λ/Λ) , (m/Λ) . The exact relations hold between the infrared fixed point “magnetic” soft masses and the ultraviolet fixed point “electric” soft masses, and correspond to a duality mapping for soft terms. We briefly discuss implications of these results for the vacuum structure of these theories.

Recent years have seen enormous progress in our understanding of strongly coupled supersymmetric gauge theories [1, 2]. In particular, a large class of models have “dual” descriptions which are weakly coupled in terms of “magnetic” degrees of freedom in the deep infrared. It is natural to attempt to extrapolate these supersymmetric results to non-supersymmetric theories by adding soft masses for the superpartners in order to decouple them [3, 4, 5, 6, 7, 8, 9]. As a modest first step towards the decoupling limit, one can study the response of the theory to soft masses m much smaller than the dynamical scale of the theory Λ . This is also of considerable interest to models where some of the Standard Model fields arise as composites of elementary “preons”. If the preon soft masses are known, what are the soft masses of the composite states? The difficulty in addressing these simple questions is that scalar soft terms are given by manifestly non-holomorphic terms in the bare Lagrangian

$$\mathcal{L}_{soft} \supset \int d^4\theta \theta^2 \bar{\theta}^2 m^2 \phi^\dagger e^V \phi \quad (1)$$

and so the usually powerful constraint of holomorphy can not be used to analyse this type of soft breaking in theories with $N = 1$ supersymmetry. Nevertheless, in this letter we will show that the leading contribution to the soft masses for the “magnetic” fields can be computed exactly in terms of the soft masses for the original fields, with uncalculable corrections suppressed by powers of (m/Λ) . These results are possible due to an interpretation of scalar soft masses as auxiliary components of the vector field of an anomalous background $U(1)$ gauge symmetry [10]. In ref. [10], this symmetry was exploited in perturbation theory, allowing high-loop supersymmetry breaking results to be obtained from lower loop *supersymmetric* computations. Here we show that the same symmetry can be useful for computing soft masses in strongly coupled theories.

For simplicity, we will consider (asymptotically free) SUSY gauge theories with a simple gauge group, softly broken by a gaugino mass m_λ and universal scalar masses m^2 . The extension of our methods to include several group factors as well as arbitrary scalar masses will be obvious. Begin by considering the exactly supersymmetric limit. The bare lagrangian with ultraviolet cutoff μ_{UV} is

$$\int d^2\theta S(\mu_{UV}) W^2 + \text{h.c} + \int d^4\theta F(S(\mu_{UV}) + S^\dagger(\mu_{UV})) Q^\dagger Q \quad (2)$$

where “ $Q^\dagger Q$ ” stands for $Q^\dagger e^V Q$. The μ_{UV} dependence of S, F is dictated by the Wilsonian renormalization group, which requires that the low energy physics stay fixed as the ultraviolet cutoff μ_{UV} is varied. It is well-known that S only changes at 1-loop [11], whereas F runs at all orders in perturbation theory:

$$\frac{dS}{d \ln \mu_{UV}} = -\frac{b}{8\pi^2} \quad (3)$$

$$\frac{d \ln F}{d \ln \mu_{UV}} = \gamma(S) \quad (4)$$

where γ is the supersymmetric anomalous dimension.

Now consider the reparametrization $Q \rightarrow \sqrt{Z}Q$. By the rescaling anomaly [12, 13] the new lagrangian is

$$\int d^2\theta (S(\mu_{UV}) + \frac{T}{8\pi^2} \ln Z) W^2 + \text{h.c.} + \int d^4\theta Z F(S(\mu_{UV}) + S^\dagger(\mu_{UV})) Q^\dagger Q. \quad (5)$$

Where T is the total Dynkin index of the matter fields Q . After relabelling $S(\mu_{UV}) + (T/8\pi^2) \ln Z \rightarrow S(\mu_{UV})$, we can start over again with the following bare lagrangian

$$\int d^2\theta S(\mu_{UV}) W^2 + \text{h.c.} + \int d^4\theta Z F(S(\mu_{UV}) + S^\dagger(\mu_{UV}) - \frac{T}{4\pi^2} \ln Z) Q^\dagger Q. \quad (6)$$

and treat S and Z as independent parameters. The theory defined by eqn.(6) is invariant under the transformation

$$Z \rightarrow Z \chi \chi^\dagger, \quad Q \rightarrow Q/\chi, \quad S(\mu_{UV}) \rightarrow S(\mu_{UV}) + (T/4\pi^2) \ln \chi. \quad (7)$$

Notice that the “physical” coupling $\text{Re}S - (T/4\pi^2) \ln Z$ is invariant.

We can now consider the situation where $S(\mu_{UV})$ and Z are respectively promoted to chiral and real vector superfields. For us this just means that these quantities have non vanishing θ^2 and $\theta^2 \bar{\theta}^2$ components, corresponding to soft gaugino and squark masses. Notice that, when S and Z are promoted to superfields, the bare lagrangian of eqn.(6) still defines a cut-off independent low energy theory also including insertions of soft terms. This follows from simple power counting. Apart from a cosmological constant term, no new divergences are generated. Indeed, these would have to involve covariant

derivatives acting on S and Z , but there is no local counterterm of this type also involving the physical fields.

Now that S and Z are superfields the above invariance becomes an abelian background $U(1)_A$ gauge symmetry. Physical quantities have to be $U(1)_A$ and RG invariant. The parameter space of the theory is described by the only $U(1)_A$ and RG invariant object that can be formed with S and Z

$$I \equiv \Lambda_h^\dagger Z^{2T/b} \Lambda_h \quad (8)$$

where $\Lambda_h = \mu_{UV} e^{-8\pi^2 S/b}$ is the holomorphic dynamical scale. The θ^0 component of I gives the “physical” strong scale $[I]_{\theta=\bar{\theta}=0} = \Lambda^2$ [14]. As we show immediately below, the θ^2 and $\theta^2\bar{\theta}^2$ components are related to the UV fixed point limits of the gaugino mass m_λ and the squark mass m_Q^2 respectively:

$$\begin{aligned} [\ln I]_{\theta^2} &\equiv \frac{16\pi^2}{b} m_g = \lim_{\mu_{UV} \rightarrow \infty} \frac{16\pi^2}{b} \left(\frac{m_\lambda}{g^2} \right) \\ [\ln I]_{\theta^2\bar{\theta}^2} &= \frac{2T}{b} [\ln Z]_{\theta^2\bar{\theta}^2} \equiv -\frac{2T}{b} m^2 = -\frac{2T}{b} \lim_{\mu_{UV} \rightarrow \infty} m_Q^2 \end{aligned} \quad (9)$$

As a first step in making these identifications, we show that in the deep UV $\mu_{UV} \rightarrow \infty$, the θ^2 and $\theta^2\bar{\theta}^2$ components of F vanish and hence make no contribution to soft terms. By dimensional analysis and invariance under the anomalous symmetry the wave function has the form $F = F(\mu_{UV}^2/I)$ (note that $-b/8\pi^2 \ln \mu_{UV}^2/I = S + S^\dagger - T/4\pi^2 \ln Z$ is just the argument of F in eqn.(6)). Therefore we have

$$[\ln F]_{\theta^2} = -\frac{1}{2} \frac{d \ln F}{d \ln \mu_{UV}} [\ln I]_{\theta^2} = -\frac{8\pi^2}{b} \gamma(\mu_{UV}) m_g, \quad (10)$$

$$\begin{aligned} [\ln F]_{\theta^2\bar{\theta}^2} &= -\frac{1}{2} \frac{d \ln F}{d \ln \mu_{UV}} [\ln I]_{\theta^2\bar{\theta}^2} + \frac{1}{4} \frac{d^2 \ln F}{d \ln^2 \mu_{UV}} |[\ln I]_{\theta^2}|^2 \\ &= \frac{T}{b} \gamma(\mu_{UV}) m^2 + \left(\frac{8\pi^2}{b} \right)^2 \dot{\gamma}(\mu_{UV}) m_g^2 \end{aligned} \quad (11)$$

where $\dot{\gamma} = d\gamma/d\ln\mu_{UV}$. As $\mu_{UV} \rightarrow \infty$, the theory becomes free, $\gamma, \dot{\gamma} \rightarrow 0$, and $[\ln F]_{\theta^2}, [\ln F]_{\theta^2\bar{\theta}^2}$ both vanish.

We now establish the first of eqns.(9). Defining the anomalous $U(1)_A$ invariant quantity R via [11]

$$R - \frac{A}{8\pi^2} \ln R = S + S^\dagger - (T/4\pi^2) \ln ZF \quad (12)$$

(where A is the Dynkin index of the adjoint representation) the physical gauge coupling in any given scheme has the form [10]

$$R_{\text{phys}} = R + \sum_{n=0}^{\infty} c_n R^{-n} \quad (13)$$

where the c_n are scheme dependent constants (the coefficient of the leading term is fixed by the equality of Wilsonian and physical coupling at tree level). At any scale μ_{UV} , we have that $m_\lambda/g^2 = [\ln R_{\text{phys}}]_{\theta^2}$. However, as $\mu_{UV} \rightarrow \infty$, $R^{-1} \rightarrow 0$, so that only the first term in eqn. (13) matters, and the first of eqns.(9) follows trivially. The same result was discussed in refs. [15, 10]. Consider now the running squark mass given by the matter kinetic term in eqn.(6)

$$m_Q^2(\mu_{UV}) = -[\ln Z]_{\theta^2 \bar{\theta}^2} - [\ln F(\mu_{UV})]_{\theta^2 \bar{\theta}^2}. \quad (14)$$

Again, as $\mu_{UV} \rightarrow \infty$, the second term in eqn.(14) vanishes and we recover the second of eqns.(9).

Having established the physical interpretation of the various components of the $U(1)_A$ and RG invariant superfield I , we discuss the computation of “magnetic” soft masses. This will be possible since the anomalous $U(1)_A$ symmetry of eqn.(7) provides a powerful constraint on the way in which S, Z (and hence the soft masses) enter into the theory. As an example, consider $SU(N)$ SUSY QCD with $(N+1)$ flavors $Q_i, \bar{Q}_{\bar{i}}$, for the moment in the supersymmetric limit. In the deep infrared and at the origin in moduli space, this theory has a weakly coupled description in terms of the composite “mesons” $M_{i\bar{i}} = Q_i \bar{Q}_{\bar{i}}$ and “baryons” $B^i = (Q^N)^i, \bar{B}^{\bar{i}} = (\bar{Q}^N)^{\bar{i}}$ [1]. We expect that, as long as the soft masses are much smaller than the strong scale Λ , the mesons and the baryons still give a good description of the low energy theory. In particular, we expect the Kahler potential for these fields to be smooth everywhere on moduli space. Therefore we can expand it in a power series in M, B, \bar{B} around the origin. By using invariance under the flavor symmetries, under eqn.(7), and under the RG, the Kahler potential must depend on S, Z

as

$$K = c_M \frac{M^\dagger Z^2 M}{I} + c_B \frac{B^\dagger Z^N B}{I^{N-1}} + c_{\bar{B}} \frac{\bar{B}^\dagger Z^N \bar{B}}{I^{N-1}} + \dots \quad (15)$$

The effective Kahler potential K is also associated with a coarse-graining scale $\mu_{IR} < \Lambda$, and the wave function coefficients $c_{M,B,\bar{B}}$ (which depend on μ_{IR}) play a role similar to F in the UV theory. At any μ_{IR} , the soft terms for the composites are given by e.g.

$$m_M^2(\mu_{IR}) = -[\ln \frac{Z^2}{I}]_{\theta^2 \bar{\theta}^2} - [\ln c_M(\mu_{IR})]_{\theta^2 \bar{\theta}^2}. \quad (16)$$

By invariance under the ultraviolet RG and the anomalous $U(1)$, the wave functions have the form $c_{M,B,\bar{B}} \equiv c_{M,B,\bar{B}}(\mu_{IR}^2/I)$. As for the UV wave function F , the dependence of the c 's on the soft terms is determined by the RG

$$[\ln c_{M,B,\bar{B}}]_{\theta^2 \bar{\theta}^2} = \frac{T}{b} \gamma_{M,B,\bar{B}}(\mu_{IR}) m^2 + \left(\frac{8\pi^2}{b} \right)^2 \dot{\gamma}_{M,B,\bar{B}}(\mu_{IR}) m_g^2 \quad (17)$$

where, similarly as before, $\gamma = d \ln c / d \ln \mu_{IR}$ and $\dot{\gamma} = d^2 \ln c / d \ln^2 \mu_{IR}$. Now, the effective theory of mesons and baryons is free in the IR. In fact it involves one marginal Yukawa interaction $W \supset \bar{B} M B$ which goes to zero for $\mu_{IR} \rightarrow 0$. More precisely as $\mu_{IR} \rightarrow 0$, $c_{C,B,\bar{B}} \rightarrow \infty$, so that the effective coupling $\lambda_{eff}(\mu_{IR})^2 \sim 1/c_M c_B c_{\bar{B}} \rightarrow 0$ and the anomalous dimensions $\gamma_{M,B,\bar{B}} \rightarrow 0$. We conclude that at $\mu_{IR} = 0$ the c 's do not affect the soft terms in eqn. (16). We emphasize that this argument is completely analogous to the one given above for the irrelevance of F to the soft terms in the deep UV. Eqn. (17) determines a relation between soft terms and RG which is somewhat similar to the one discussed in ref. [16]. In that case the role of the invariant I was played by the messenger threshold superfield XX^\dagger .

By the above discussion, the IR fixed point value of the composites are determined by the $\theta^2 \bar{\theta}^2$ components of Z, I which are in turn related to the UV fixed point value of the squark masses as in eqn.(9). We therefore find a purely algebraic relationship between the composite soft masses in the deep IR and the squark masses in the deep UV. Using the anomalous $U(1)$ symmetry, we can seemingly control the exact soft masses for the composites, at least at the origin of moduli space! This is however true only in the limit in which the soft masses m_g and m are much smaller than the strong scale

Λ . Indeed there are $U(1)_A$ invariant terms in K which can involve the $U(1)_A$ “field strength” $W_A^\alpha = \bar{D}^2 D^\alpha \ln Z$ which is non-vanishing when $\ln Z$ has a non-vanishing $\theta^2 \bar{\theta}^2$ component. One such term is

$$\int d^4\theta \left(\frac{D_\alpha W_A^\alpha}{I} \right) M^\dagger \frac{Z^2}{I} M \quad (18)$$

Since W_A^α has positive mass dimension, however, this and all other such operators make contributions to the composite soft masses which are suppressed by powers of (m/Λ) . It is these uncontrollable operators which prevent us from taking the decoupling limit $(m/\Lambda) \rightarrow \infty$, however, their effects are power suppressed for $(m/\Lambda) \ll 1$.

We have now all the ingredients to determine the mapping of soft terms between the microscopic and macroscopic theories, up to corrections suppressed by powers of (m^2/Λ^2) :

$$m_M^2(\mu_{IR} = 0) = -[\ln \frac{Z^2}{I}]_{\theta^2 \bar{\theta}^2} = \frac{2N-4}{2N-1} m_Q^2(\mu_{UV} = \infty) \quad (19)$$

$$m_{B,\bar{B}}^2(\mu_{IR} = 0) = -[\ln \frac{Z^N}{I^{N-1}}]_{\theta^2 \bar{\theta}^2} = \frac{2-N}{2N-1} m_Q^2(\mu_{UV} = \infty). \quad (20)$$

These masses satisfy the relation $m_M^2 + m_B^2 + m_{\bar{B}}^2 = 0$. This sum rule can be inferred from the low energy theory due to the RG “focusing” effect of the Yukawa interaction $\bar{B}MB$, and could have been established without using the anomalous symmetry[7]. The symmetry is however crucial to fix the value of each mass. Notice that for $N > 2$ and for positive squark masses the baryons are tachyonic. The implications of this result for the symmetry properties of the vacuum will be discussed below. Notice also that for the special case of $SU(2)$ the baryons and the mesons coincide and have vanishing soft mass. While this result holds in the deep IR, by eqs.(16-17) we can establish how this limit is approached. This is a pure Yukawa theory for which both γ_M and $\dot{\gamma}_M$ are negative in the perturbative domain. Therefore we conclude that m_M^2 is positive at finite μ_{IR} and approaches zero as $\mu_{IR} \rightarrow 0$.

The result in eqn. (20) has a nice interpretation in terms of the anomalous $U(1)$ charges of the canonically normalized fields $\hat{M} = M/\Lambda_h$ and $\hat{B} = B/\Lambda_h^{N-1}$, (Λ_h has charge $2T/b$). In terms of the canonical fields eqn. (15) reads

$$K = c_M \hat{M}^\dagger Z^{q_M} \hat{M} + c_B \hat{B}^\dagger Z^{q_B} \hat{B} + c_{\bar{B}} \hat{\bar{B}}^\dagger Z^{q_{\bar{B}}} \hat{\bar{B}} + \dots \quad (21)$$

The charges of the composites $q_{\hat{M}}$ and $q_{\hat{B}}$ coincide with the corresponding IR/UV mass ratios of eqn. (20). Notice that, without the contribution to the soft terms from the powers of I in eqn. (15), the soft masses of the composites would just be determined in the “naive” way, by adding the masses of the constituents.

We stress that the existence of a relationship between deep UV and IR quantities is just a consequence of RG invariance. For instance, in QCD one may ask for the expression of the pion mass in terms of the fundamental parameters. It will have the form $m_\pi^2 = c \hat{m}_q \Lambda_{QCD}$, where c is a constant and \hat{m}_q is an RG invariant combination of the running quark mass $m_q(\mu)$ and gauge coupling $g^2(\mu)$. In practice, as for m_g in eqn.(9), one can define \hat{m}_q just by using the 1-loop RG in the deep UV $\hat{m}_q = \lim_{\mu \rightarrow \infty} g^p(\mu) m_q(\mu)$, where p is determined by the 1-loop β function and mass anomalous dimension. The striking feature of our case, with respect to QCD, is that we can calculate the analogue of the coefficient c .

Another example is given by $Sp(k)$ gauge theory with $2k+4 = 2N_F$ chiral multiplets L_i in the fundamental representation ($SU(2)$ with 3 flavors is just the special case $k = 1$). The low energy description involves the antisymmetric meson field $V_{ij} = L_i \epsilon L_j$ with a superpotential $W_{\text{conf}} = \text{Pf} V / \Lambda_h^{2k+1}$ [17]. By adding soft terms the resulting mass for the meson is (from now on it is understood that the LHS and RHS soft masses are the IR and UV fixed point values respectively)

$$m_V^2 = \frac{2k-1}{2k+1} m_L^2 \quad (22)$$

Notice that $\text{Pf} V \sim V^{k+2}$ is an irrelevant operator in the low energy theory for $k > 1$. In this case $\gamma_V(\mu_{IR}^2/I)$ goes to zero with a power law when $\mu_{IR} \rightarrow 0$ and the “running” mass $m_V^2(\mu_{IR})$ approaches eqn.(22) equally fast.

Finally consider $SU(N)$ gauge theory with $N+1 < N_F < 3N/2$ for which the low-energy description is in terms of a dual “magnetic” theory with gauge group $SU(N_F - N)$. The magnetic theory contains an elementary meson $M_{i\bar{j}}$ and N_F flavors of dual quarks $q^i, \bar{q}^{\bar{i}}$ in the fundamental representation of $SU(N_F - N)$. The $U(1)_A$ charge of the canonically normalized meson $\hat{M}_{i\bar{j}} = M_{i\bar{j}}/\Lambda_h$ is just $q_{\hat{M}} = q(Q_i \bar{Q}_{\bar{j}}/\Lambda_h) = 2(3N - 2N_F)/(3N - N_F)$. The charge of the dual quarks simply follows from the invariance of the tree level magnetic superpotential $W_{\text{magn}} = \bar{q}^{\bar{i}} M_{i\bar{j}} q^j / \Lambda_h$ (or by matching the baryons in the two theories $b = q^{N_F - N} = Q^N / \Lambda_h^{2N - N_F} = B / \Lambda_h^{2N - N_F}$). We thus obtaining

the soft masses of the magnetic theory

$$m_M^2 = 2 \frac{3N - 2N_F}{3N - N_F} m^2 \quad m_{q,\bar{q}}^2 = -\frac{3N - 2N_F}{3N - N_F} m^2. \quad (23)$$

Again the “baryons” q, \bar{q} are tachyonic for all theories in the free magnetic phase $N_F < 3N/2$. For $3N/2 < N_F < 3N$ the theory is in an interacting non-Abelian Coulomb phase. Here we cannot apply our method in an obvious way since there are no points where the theory is free. It is interesting that all the magnetic soft masses vanish at the boundary between the free magnetic and conformal windows, $N_F = 3N/2$.

Notice that the normalization of the magnetic quarks q, \bar{q} is arbitrary, and that in ref. [1] a scale μ was introduced to give proper dimension to the superpotential: $W_{\text{magn}} = M_{i\bar{j}} q^i \bar{q}^{\bar{j}} / \mu$. Correspondingly the holomorphic scales in the electric and magnetic theory are related by [1] $\Lambda_h^b \tilde{\Lambda}_h^{\tilde{b}} = \mu^{N_F}$ (where the tilded quantities refer to the magnetic theory). In our derivation we have fixed $\mu = \Lambda_h$, but our results do not depend on that choice. Indeed one could have argued as follows. With the normalization of ref. [1] the dual quarks have charge -1 under the anomalous $U(1)_A$ (μ is neutral and $M_{i\bar{j}} = Q_i \bar{Q}_{\bar{j}}$). As we did for the electric theory in eqn. (6), we can define a dual wave function \tilde{Z} multiplying the kinetic term of the dual quarks q, \bar{q} as well as an invariant scale $\tilde{I} = \tilde{\Lambda}_h^\dagger \tilde{Z}^{2T/\tilde{b}} \tilde{\Lambda}_h$. However, since the electric and magnetic theory describe the same physics, it must be $\tilde{I} = I$. Therefore we deduce the following “duality” relation

$$\tilde{Z}^b = Z^{\tilde{b}} \quad (24)$$

which holds up to a gauge transformation which does not affect the mapping of soft terms. Eqn.(24) correctly gives the magnetic quark masses in eqn.(23). By eqn.(24) the opposite sign of electric and magnetic squark masses is a reflection of duality between IR and UV free theories. Finally, by considering $[\ln \tilde{I}]_\theta^2 = [\ln I]_\theta^2$, a similar duality is obtained for gaugino masses. More precisely one gets (notice again the flip in sign)

$$\lim_{\mu_{IR} \rightarrow 0} \left(\frac{m_\lambda}{\tilde{b}g^2} \right)_{\text{magn}} = \lim_{\mu_{UV} \rightarrow \infty} \left(\frac{m_\lambda}{bg^2} \right)_{\text{el}}. \quad (25)$$

Our results for the soft masses of composite and “magnetic” fields have obvious implications for composite model-building in theories where supersymmetry breaking is communicated to the “preon” fields at a scale higher

than the dynamical scale Λ (for instance by supergravity mediation). Clearly one must check that e.g. none of the composite squarks obtain negative soft masses.

Next we consider the vacuum structure of these theories, beginning with the $SU(N)$ theories with $N + 1 \leq N_F \leq 3/2N$ flavors and $N > 2$. In the supersymmetric limit, these theories have a moduli space of vacua, and we are interested in how the soft breaking effects lift this vacuum degeneracy. In all these theories, for positive squark masses in the deep ultraviolet, the “meson” fields get positive soft masses while the “baryonic” fields get negative soft masses. The origin of moduli space is therefore unstable, and some of the mesons or baryons must have non-vanishing vevs in the true vacuum. Note that our method only gives us information on the form of the potential close to the origin, since far from the origin operators with higher powers of meson and baryon fields (which we have no control over) are unsuppressed, and therefore we can not determine the location of the true vacuum even for small soft breakings. Nevertheless, establishing the instability of the origin has important consequences, since in these theories all points on the moduli space away from the origin break vector-like symmetries. If any baryonic fields obtain vevs baryon number is broken, and if all the baryons vevs vanish, there is no point on the quantum moduli space where $M_{i\bar{j}} \propto \delta_{i\bar{j}}$ and so $SU(N_F)_V$ is broken. This is to be contrasted with the non-supersymmetric theory obtained by decoupling the scalars, where a general theorem [18] shows that vector-like symmetries are never broken. It is easy to argue that the broken vector-like symmetries are restored for squark masses larger than a (finite) critical value. Squarks of mass $m^2 \gg \Lambda^2$ can be integrated out of the theory, generating higher dimension operators suppressed by $1/m^2$ in the non-supersymmetric low energy theory. These operators can at most correct the spectrum of states in the low energy theory by $O(\Lambda^2/m^2)$. Since all the scalar states in the non-supersymmetric theory get masses of $O(\Lambda)$ (with the exception of Goldstone bosons associated with chiral symmetry breaking), there are no scalars which can be brought down to zero mass due to the $O(\Lambda^2/m^2)$ corrections and there is therefore no candidate for the Goldstone boson of a broken vector-like symmetry. Therefore, the vector-like symmetries must be exactly restored above a finite critical squark mass $m_*^2 \sim \Lambda^2$, and a phase transition must separate the nearly supersymmetric and non-supersymmetric theories.

For $Sp(m)$ theories with $2m + 2$ chiral multiplets, the soft mass of the

mesons is positive and the origin of moduli space is at least a local vacuum. At this point, the fermionic mesons are massless bound states of a massless quark and a massive squark, the binding energy exactly cancelling the squark mass. This provides a rigorous counter-example to the the “persistent mass condition” of [19, 20].

In conclusion we remark that, while we have illustrated our ideas with two specific examples, our technique for computing soft masses can clearly be applied in any asymptotically free supersymmetric theory where the theory in the deep infrared is known and is weakly coupled, as in all s -confining [21] or magnetic free theories.

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